Inferences and Proofs
Marseille
May 31 - June 1st 2016
Aix-Marseille University

Abstracts

May 31
Morning session

09:30 / 10:30
Dag Prawitz - Stockholm University
The seeming interdependence between the concepts of inference and proof

A proof is naturally explained as a chain of valid inferences, but conversely, when explaining the notion of validity in this context, one naturally refers to the notion of proof. My talk surveys some proposed ways of escaping this circle based on Gentzen's ideas about natural deduction and intuitionistic ideas about the concept of proof originating from Heyting.

10:50 / 11:50
Cesare Cozzo - "La Sapienza" University of Rome
Cogency and context

A cogent inference compels us to accept its conclusion, if we accept its premises. Should a philosophical explanation of cogency consider the fact that inferences occur in a context? What kind of context can be relevant to an explanation of cogency? What is the relevant notion of inference? Does cogency in some sense depend on context? If it does, in what way does context bear upon the compelling force of an inference? These are the questions I will address.

11:50 / 12:50
Peter Schroeder-Heister - Tübingen University
Beyond Logic: Two concepts of proof-theoretic semantics

I compare the most common approach to proof-theoretic semantics, which is based on closed proofs as its starting point, with an approach based on the primacy of the hypothetical. This issue goes beyond logic in the narrower sense, touching on issues that extend the pattern of inversion to general rules systems and also call for a refined treatment of negation and denial.

May 31
Afternoon session

14:00 / 15:00
Kosta Došen - Belgrade University and Mathematical Institute
Gödel on deduction

Gödel's views concerning deduction will be examined in the light of the unpublished notes for the elementary logic course he gave at the University of Notre Dame in 1939. In this course he formulated a natural deduction system and spoke about the advantages of this manner of presenting logic.

References

[A.&D. 2016] M. Adzic and K. Dosen, Gödel's Notre Dame course
preprint
http://www.mi.sanu.ac.rs/∼kosta/ADgoedndcourse.pdf
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[D.&A. 2016] K. Dosen and M. Adzic, Gödel on deduction
preprint
www.mi.sanu.ac.rs/∼kosta/DAgoedded.pdf
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www.mi.sanu.ac.rs/∼kosta/DAgoednatded.pdf
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15:00 / 16:00
Gabriella Crocco
Aix-Marseille University
Informal rigour and proofs

Starting from Georg Kreisel's concept of informal rigour, the purpose of this paper is to discuss various hints concerning the notion of proof given by Kurt Gödel in his published and unpublished work and to compare them with the contemporary discussion about the notion of the epistemic power that a proof should convey in order to compel one to accept its conclusion. In his 1951 paper, Gödel seems to suggest that there are three essential characteristics of a formal proof (i.e. a proof that can be performed by a Turing machine): finitude, locality and independence from meaning. He seems also to suggest that at least the last two of them have to be dropped out in the case of subjective mathematics (mathematics that can be afforded by

a human being through the notion of proof). Is this Gödelian notion of a non-formal rigorous proof compatible with the notion of inference? We will try to give an answer to this question.

16:20 / 17:20
Gabriele Usberti
Siena University
Inference and epistemic transparency

In his paper *Explaining deductive inference* Prawitz states what he calls "a fundamental problem of logic and philosophy of logic": the problem of explaining "why do certain inferences have the epistemic power to confer evidence on the conclusion when applied to premisses for which there is evidence already?". In this paper, I suggest a way of articulating, and partly modifying, the intuitionistic answer to this problem in such a way as to satisfy a requirement I argue to be crucial for an intuitionistc theory of the meaning of the logical constants: the requirement that evidence is epistemically transparent.

17:20 / 18:20
Nissim Francez - Technion Israel Institute of Technology
Natural deduction for two connexive logics

I propose two natural deduction systems $N^{\neg r}$ and $N^{\neg l}$, for non-classical interactions of a certain kind between negation and implication, that can be seen as variants of connexive logics. These variations are inspired by a certain use of negation and implication in natural language. I propose the natural deduction systems $N^{\neg r}$ and $N^{\neg l}$ as meaning-conferring proof-systems, not appealing to any many-valued model theory as a semantics. The model-theory is used mainly as an auxiliary tool for establishing non-derivability, for example of some classical formal theorems (or, more generally, classical derivability claims) that are not provable (not derivable) in $N^{\neg r}$ and $N^{\neg l}$. The relation between implication and negation in the system $N^{\neg r}$ is similar to the one by Cantwell and one by Cooper, the former unaware of the latter. The system $N^{\neg l}$ seems to be new.

18:20 / 19:20
Miloš Adžić - Belgrade University
Gödel on absolute proof and the logic of concepts

In the course of his conversations with Hao Wang, Kurt Gödel discussed, among others, two very interesting and related topics. One is the notion of absolute proof and the other one is his intensional theory of concepts, which should, when developed, secure further his conceptual realism. Although

these remarks were made during the early 1970s, the ideas were in Gödel's mind for a long time already. His work on Russell's logic can be seen as considering various proposals for the realistic theory of concepts, finding them at the end all unsatisfactory. On the other hand, he suggests that we look for the definitions of the notions of definability and provability, which are absolute in the sense of Turing's definition of the notion of computability. Unfortunately, what he says there about the notion of absolute provability is very brief and mostly limited to the case of provability in set theory. It seems that although Gödel paid a great deal of attention to these issues, he did not reach a conclusion that he would find completely satisfactory. Some of the remarks Gödel made about these topics are taken as an inspiration, and a way to interpret them is proposed from the standpoint of contemporary logic and in particular general proof theory. For instance, the importance Gödel assigns to rules of inference in logic and the need for intensional considerations regarding the concept of proof are reflected in categorial proof theory, where we are interested in identity criteria for proofs and where taking a purely extensional point of view on the matter makes the question trivial.

June 1st

09:30 / 10:30
Per Martin-Löf - Stockholm University
Judgement and inference

When semantically justifying, or validating, a rule of immediate inference, one invariably begins by saying: assume the premises, whereupon follows an explanation serving to justify the conclusion. Since both the premises and the conclusion of an inference are judgements, these assumptions are judgements in contradistinction to the propositions which are assumed in natural deduction. Göran Sundholm has called them epistemic assumptions in his paper *Implicit epistemic aspects of constructive logic*, Journal of Logic, Language, and Information 6, 191 - 212, 1997. Being judgements rather than propositions, there arises the question: of what mood (modality in Kant's idiom) are they? Just as, when making an assumption in natural deduction, we are not merely considering (Russell's idiom) a proposition, in the case of an epistemic assumption, we are not merely considering it, that is, considering what it means. So the problematic mood, in Kant's terminology, is excluded. There remains the question whether it is assertoric or apodictic. This is tantamount to the question: are we assuming that the assumption judgements have been made or are we, which would be stronger, assuming that they have been demonstrated? I shall opt for the first alternative, that is, the assertoric mood, thereby avoiding an otherwise disturbing circularity in the definition of the notion

of demonstration.

10:50 / 11:50
Göran Sundholm - Leyden University
***Grundlagen*, §17**

I consider the difference between propositions and judgemental contents, as well as the difference between inference, from judgement(s) to judgement, and consequence, be it logical or not, among antecedent and consequent propositions. Furthermore I resist the customary (Bolzano) reduction of inferential validity to the (logical) holding of consequence. Against this background, after a close reading, Frege's treatment in GLA, § 17, drawing also upon relevant passages in the *Logik* of the 1880's, *Grundlagen der Geomterie*, II:nd series, part 3, and *Gedankengefüge*, is found wanting.

11:50 / 12:50
Luca Tranchini - Tübingen University
Proof, meaning and paradoxes: Some reflections

The aim of the paper is to show how proof-theoretic semantics (PTS) provides the tools to make sense of the idea that a paradoxical sentence, though semantically defective, is nonetheless meaningful, and it is in virtue of our understanding its meaning that we classify it as paradoxical. We will first show that PTS provides the means to distinguish between paradox and other kinds of semantic defectiveness. We will then show that the PTS characterization of paradox lends itself to be recast using the notion of isomorphism from categorial proof-theory, thereby providing further evidence for taking this notion as a possible explanans of identity of meaning. Finally, we will argue that the watershed between paradoxical and non-paradoxical meaning explanations bears strong analogies to the one between realism and anti-realism.